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STUDENTS' MATHEMATICAL KNOWLEDGE CONSTRUCTION PATTERNS IN PROBLEM SOLVING CONTEXT

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ABSTRACT

The study investigated the nature of knowledge construction patterns (or thinking schemata) students manifested in solving a non-routine problem, distribution of students according to knowledge construction patterns and their success rates when grouped according to relatedness of course to mathematics and degree of exposure to mathematics. There were 217 respondents from different degree programs and year levels who were given carefully selected non-routine problem which they solved in one hour.

Results showed there exist seven different knowledge construction patterns with varying degrees of success. The proportion of respondents exhibiting successful knowledge construction patterns accounts for 12.9%, partially successful, 48.93% and not successful, 39.17%, thus indicating that large proportion of students need appropriate training in solving non-routine problems to improve their solving abilities. The proportions of respondents manifesting similar knowledge construction patterns across groups, as well as their success rates, are not significantly different. The results indicate that motivation and exposure to routine mathematics are not factors that differentiate between those with successful knowledge construction patterns and those with less successful ones and that students' mathematics learnings are independent of the development of productive thinking schemata and solving abilities.

Keywords: *problem solving, mathematical knowledge construction patterns and thinking schemata*

INTRODUCTION

Knowing how likely students think, or construct knowledge, especially in dealing with problem situation is definitely a relevant topic for exploration in education for this major reason: if teachers have knowledge on how students tend to solve a particular type of problems then they are better able to frame problem situations that bring out students' patterns of thinking and thereafter be able to redirect less productive thinking patterns to more productive ones. They can also design activities that truly serve as battle ground for students' development of problem solving skills and correct thinking schemata.

Over time, the importance of problem solving in mathematics education as both means of learning and doing mathematics is gaining impetus among educators worldwide pointing to many learning benefits it gives to learners. It is argued that "by learning problem solving in mathematics, students acquire ways of thinking, habits of persistence and curiosity and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages (NCTM as cited by Daane, 2004). Also, learning to solve problems is the principal reason for learning mathematics (Wilson *et al.*, 2000) and the main reason for learning all about Math is to become better problem solvers in all aspects of life (Deb, 2013). While educators and researchers worldwide believed that problem solving is the heart of mathematics education, results of researches on students' performances in problem solving cast doubt whether traditional mathematics education, which consisted mostly of drilling students to master computational skills and acquire competence on factual and procedural knowledge, is helping students develop desirable problem solving schemata that are flexible and adaptive to new situations. In some studies, it was claimed that many students in mathematics courses who are expected to perform well in problem solving were unsuccessful in solving non-routine problems. The studies conducted by Asman and Markowitz (2001) and Higgins (1997) as cited by Arslan and Altun (2007) showed that despite long years of instruction, many students have insufficient aptitude and confidence to approach mathematical problems, especially non-routine ones, in a successful way. These results appear to support the claim of Polya (1962), as cited by Yeap (2008), that solving routine problems such as those that require the mere applications of given formulas do not contribute to the mental development of the students.

Several local studies revealed that it is in problem solving where most students fail to perform well. Among these is the study of Rabbacal (2013) which found that only one, out of 18 third year education students majoring in mathematics, is able to solve a non-routine problem correctly. The majority are in the apprentice level of solving ability, or those who manifest only partial understanding of the problem and employ somewhat reasonable strategies but fail to arrive at the correct answer.

The consistent findings of researches showing poor performance of students in problem solving appeared to have motivated many countries including USA to integrate problem solving in their mathematics curricula.

In particular, a country that can be said as having given much importance to problem solving and emphasized it in its mathematics curricula is Singapore. According to Clark (2008), since 1992, the Singapore Ministry of Education has put mathematics problem solving as central to mathematics learning which involves the acquisition and application of mathematical concepts and skills in a wide range of situations, including non-routine, open-ended and real world problems. Such initiative made Singapore a top performing nation in science and mathematics as shown in their scores in the Trends in International Math and Science Study (TIMSS) comparison assessments where it emerged as top one in 1995, 1999, 2003 and as top three in 2007.

In the case of the Philippines, while several changes in basic mathematics education curricula in both elementary and secondary have taken place almost at the same time when Singapore rewrote its mathematics curricula in the early 90s, it appeared there have been no significant impacts of those changes in view of the Philippine performances in international mathematics and science competency examinations. After rewriting its basic education curricula in the early 90s under the so called Secondary Education Development Program (SEDP), the Philippines participated in the TIMMS in 1999 and 2003 but the results were dismally disappointing. In 1999, the Philippines emerged third from the bottom among 48 participating countries in both mathematics and science scoring a total of 345 points in math compared to Singapore's 604 and besting only Morocco (337) and South Africa (275). In 2003, the same result yielded with Philippines earning a total score of 358 which is way below international average of 495 and besting only Morocco (347) and Tunisia (339) (eduphil.org). Since then, the Philippines never participated again in such international competency examination. It is noted, though, that while changes in Philippine basic mathematics curricula was observed, the fact remains that the focus of education has not been on problem solving but on the usual mastering of facts and procedural knowledge as well as on acquiring computational skills.

In the light of the above information, this study aims to determine what kind of problem solving schemata the students manifest in solving a non-routine problem or real life problem situation. It is the belief of the researcher that knowledge on how students tend to solve a particular type of problem could be an important key in enabling teachers to come up with better ways of helping students improve their solving skills and develop effective thinking patterns. Further, if ability to solve real life mathematical problems is a gauge of whether existing school mathematics are imbuing relevant and useful knowledge, skills and mental frames that enable a person to improve his life situation, then sustainable efforts to monitor the students' problem solving performances via research are in order.

Objectives

The study aims to determine the knowledge construction patterns of the students when exposed to a non-routine problem solving situation in order to identify those thinking patterns that are less productive and those that are more productive and to determine the nature of the construction patterns exhibited by the different groups of students and the variances of success rate of the different patterns.

METHODOLOGY

Both qualitative and quantitative research method were used. The qualitative part consisted of identifying and describing the different knowledge construction patterns and translating them to visual images by way of diagrams. The quantitative part consisted of determining the distributions of respondents with respect to knowledge construction patterns and to success categories.

The respondents of the study involved 217 students from different degree programs and year levels who comprised the researcher's total number of students from his seven classes during second semester of SY 2012-2013.

They were grouped according to relatedness of course to mathematics and extent of exposure to mathematics. Freshman students enrolled in Agricultural Engineering and Applied Statistics comprised the group with mathematics related courses while education students majoring in Filipino and Library comprised the group with non-mathematics related courses. Finally, education students majoring in mathematics and belonging to the upper years comprised the group with more extensive experience in mathematics.

The students were given an hour to solve this carefully selected non–routine or real life problem situation: *Juana bought a necklace from a jewelry shop at PhP750.00 and sold it to Maria at PhP850.00. When Juana learned that her friend, Petra, is looking for a necklace which is exactly the same as the one she sold to Maria, she bought back the necklace at PhP950.00 and sold it to her friend at PhP1050.00. Now, did Juana gain, or lose, or neither gain nor lose from the transactions? If she either gained or lost from the transaction, then how much?*

The problem given was pretested to a group of students not included in the study to determine whether it can elicit among the respondents varied exploratory attempts and varied knowledge construction patterns which are characteristics primarily considered in choosing a problem. The group tested showed varied exploration attempts and seven different thinking patterns, a number the researcher believed enough to consider the problem a good tool in generating data. The choice of the problem is consistent with what the mathematician and researcher, Schoenfeld (1994) said that good Math problems are those that are capable of being extended for exploratory attempts by the solvers. The problem given is considered a power problem and considering the amount of time expended to solve it, answers generated can be considered close approximates of how likely students analyze and respond to similar type of problems. The problem, although appearing simple in substance, contains logical subtleties that may discriminate between those who have more developed problem solving schemata and can analyze problems correctly and those who are still struggling to become successful problem solvers. As in other mathematical researches conducted that made use of a single problem to analyze students solving abilities, the single problem method adopted was but this time it is applied in determining the nature of knowledge construction patterns elicited. The researcher found the given problem enough tool to gather data as it is capable of eliciting divergent knowledge construction patterns. It likewise represents those myriad of problem types that elicit responses which enable people to determine those who can analyze problem situations properly and those who are exhibiting alternative analyses.

Before solving the problem, the respondents were instructed to provide complete solutions and/or explanations for their answers so as to provide a clue in identifying the nature of knowledge constructions patterns used to interpret and solve the problem. After collecting the outputs, the solutions were sorted and classified according to the final answers given and corresponding solutions provided. Each group of answers and solutions were analyzed carefully to identify the nature of knowledge constructions involved.

The different knowledge construction patterns exhibited within each group and across groups were coded for easy reference, described in detail and traced via diagrams to provide visual images of the construction patterns pursued by the solvers. These images were presented to the students for validation of the construction patterns used to solve the problem.

Finally, the number of students exhibiting the different patterns was tallied in order to determine which pattern is used by majority and minority of the students. The solution patterns correspond to the knowledge construction patterns of the students.

In order to represent the various knowledge construction patternsin diagrams, the following notations were used: E1 stands for the initial expense of 750, E2 for the second expense of 950, S1 for the first sale of 850, S2 for the second sale of 1050, G for a gain of 100, L for the additional expense of 100, E1-S1 for the first transaction, S1-E2 for the second transaction and finally E2-S2 for the third transaction. The solution patterns represent the knowledge construction patterns of the students.

The knowledge construction patterns exhibited were compared across groups to determine if nature of course and exposure to mathematics have any association with the way students construct their knowledge when exposed to a real life mathematical problem situation.

Pearson Chi-Square and Fisher's Exact Test were used to determine whether the proportions of students exhibiting similar thinking patterns across the three groups are of different or not.

RESULTS AND DISCUSSION

The Nature of Knowledge Constructruction Patterns

The solutions and final answers given by the students range from logically correct analyses, partly reasonable, unclear, to extremely bizarre explanations. The final answers given include gains of 0, 100, 200, 300, 400, and losses of 750 and 1500.

The construction pattern yielding net gain of 0 (TPG0) are depicted in figures 1(a) and 1(b) below.

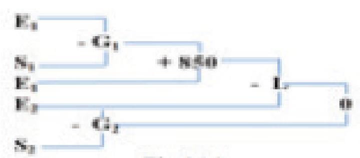

Fig 1(a)

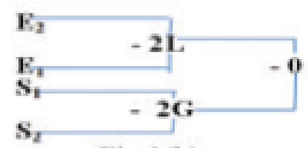

Fig 1(b)

The construction pattern in figure 1(a) says that the money yielded in the first transaction is part of the new capital of 950 that was yielded in the second transaction. But the additional expense incurred in the second transaction cancels out with the gain obtained in the third transaction thereby yielding a net gain of 0. Whereas that of figure 1(b) says that net gain is computed as the difference between increase in sale of 200 from 850 to 1050 and increase in expense equal amount from 750 to 950 which yield 0. The gap in these construction patterns lies in the failure of the students to recognize the overall existence of profit in the whole sequence of transaction.

The construction pattern yielding net gain of 100 (TPG100) as depicted in figure 2 says that in the whole transaction, there were two occasions when a gain of 100 was realized which happened in the first and third transactions while one occasion when a loss of 100 was incurred which happened in the second transaction thereby yielding a net gain of 100. The analysis in this construction pattern is partly correct but what the solvers failed to consider is the difference of 300 between the amount on hand (1050) after the final sale and the original cost (750)

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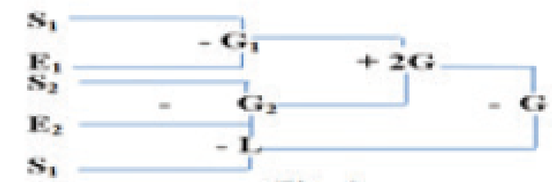

Fig. 2

Fig. 3(a)

Fig. 3(b)

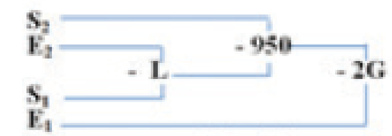

Fig. 3(c)

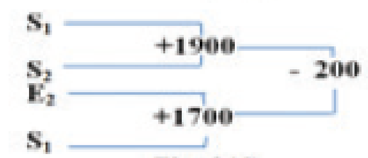

Fig. 3(d)

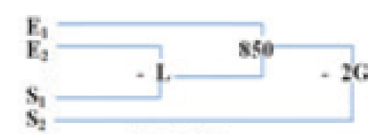

Fig 3(e)

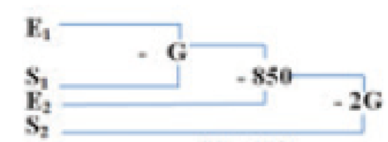

Fig. 3(f)

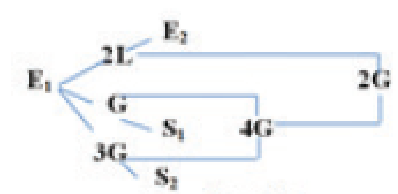

Fig. 3(g)

of the item. Subtracting the additional expense of 100 from the given difference does not only yield a profit of 100 but 200.

The group of students who gave a net gain of 200 (TPG200) exhibited seven different ways of arriving at the same answer and these are depicted in figures 3(a), 3(b), 3(c), 3(d), 3(e), 3(f) and 3(g).

The above construction patterns are the correct ones. These patters affirmed one of the strengths of mathematics which is, however, a problem is solved, if all information are accounted properly and the solution steps knit together logically, then the correct answer will be yielded. What is common in the above construction patterns is that the solvers recognized the overall increasing value of the item in the whole sequence of transactions and they have correctly accounted the effect of the additional expense incurred.

The construction pattern yielding a net gain of 300 (TPG300) (Fig. 4) says that gain in the first transaction and loss in the second transaction cancels out each other bringing the net expense to 750 which is 300 lower than the money on hand of 1050 after the final sale. This construction pattern is partly correct but what the solver failed to recognized is that the initial gain of 100 is part of the amount yielded after the final sale of 1050 and that the additional expense of 100 should not be subtracted from this initial gain but from the amount of the final sale to yield the correct profit of 200.

The construction pattern yielding a net gain of 400 (TPG400) (Fig. 5) indicated that the results of the first two transactions cancel out each other yielding the net expense of 750. The gross revenue from the gain of 100 in the third transaction and the money on hand of 1050 after the final sale amount to 1150 which is 400 greater than the net expense of 750. The big gap in this construction pattern is that the solver treated the profit in the first and third transactions of 100 each as money yielded separate from the amount generated after the final sale of 1050 resulting to double accounting of profits and creating artificially bloated income.

Fig 4

Fig. 5

The more bizarre construction patterns include those that gave losses of 750 (TPL750) and 1500 (TPL1500). Students who gave a loss of 750 showed the same solution pattern shown in figure 1(a), except that in this case, they concluded that there is a net loss of 750. The glaring gap in this construction pattern is, among others, the solvers' accounting of the cost of the item as a loss. If there is no gain, the result is breakeven and not a loss. The construction pattern yielding net loss of 1500 is the same as TPL750 but indicated that because there is no gain plus the fact that the necklace is not with the seller then the total loss is 1500. The gap in this construction pattern inherits that of TPL750 plus the alternative interpretation of considering the sold item as a loss in itself which should not be the case.

Distribution of Respondents With Respect to Knowledge Construction Patterns

Table 1 shows that only 12.9% manifested successful construction patterns (TPG200) and the large proportion of 87.1% manifested other construction patterns that are, in varying degrees, less successful. Among the less successful solution patterns, the one yielding a gain of 100 (44.7%) appeared dominant, followed by the one yielding a gain of 0 (18.43%) and by unclear articulation of solutions (TPUS, 17.97%).

Table 1. Distribution of students with respect to knowledge construction patterns

Solution Pattern	Frequency	Proportion (%)
TPG 0	40	18.43
TPG100	97	44.7
TPG200	28	12.9
TPG300	7	3.23
TPG400	2	1.38
TPL750	3	0.46
TPL1500	1	17.97
TPUS	39	100.00
Total 217		

Successful knowledge construction patterns in the context of the problem given are those that yielded the correct answer of PhP200, accounted all information properly and came up with solution steps logically. Partly successful knowledge construction patterns are those that yielded answers of PhP100 or PhP300, failed to account some information properly, and Table 1. Distribution of students with respect to knowledge construction patterns came up with logical but insufficient solution steps. Unsuccessful knowledge construction patterns are those that yielded other answers aside from 100, 200 and 300; failed to account some information properly, and came up with alternative logical solution steps which are not implied in the problem situation given.

The existence of the different thinking patterns shows that people think and construct knowledge in diverse ways, a result that supports the claim in constructivist theory that knowledge construction is personal (Hein, 1991) and may be varied depending on the maturity and sophistication of the learner. There are patterns that are more productive albeit very few; others that are less productive and they comprise the bulk of the population, a fact that suggests many students need some help and trainings. It is quite alarming that a considerable proportion of the students (17.97%) cannot clearly articulate their solutions, a situation that is hardly expected among college students except if they lack exposure to the above type of problem solving activities such that their abilities to make coherent and comprehensible articulations of mathematical ideas have not been developed. The data in Table 2 showed many college students still do not possess productive problem solving schemata and effective mathematical communication skills. How to make less successful knowledge construction patterns successful through educational intervention is a challenge.

In determining whether the distributions of the two groups of students with respect to knowledge construction patterns are different or not, Pearson Chi-Square and Fisher's Exact Test were used. At 0.05 level of significance (2-sided), both tests showed that the distributions are not significantly different, indicating that the proportions of students across groups who think in similar ways are not far

apart.

Although it is noted that the proportion of students who have successful construction patterns is higher among those with math related courses (15.38%) than those with non-math related courses (8.57%) and also that the former group registered smaller proportion (14.29%) ofstudents who are not able to clearly articulate their solutions compared to that of the latter group (30%), yet statistically, the differences in representation as a whole are not significant. This indicates that the observed differences are not enough to claim for their dissimilarity. Thus, the hypothesis that the mathematics subjects with

Table 2. Comparative distributions of the two groups of freshman students with respect to knowledge construction patterns problem solving thinking schemata is concerned

Solution Patterns	Math related courses	Non-math related courses
TPG 0	16 (17.58%)	12(17.14%)
TPG100	40 (43.96%)	30(42.86%)
TPG200	14(15.38%)	6(8.57%)12.9
TPG300	6(6.59%)	1(1.43%)
TPG400	2(2.2%)	0
TPUS	13(14.29%)	21(30%)
Total	19	70

Pearson Chi-square: value = 10.086, exact sig. (2-sided) = 0.063 Fisher's Exact Test: Value =9.394, exact sig. (2-sided) = 0.076

respect to knowledge construction patterns is not significantly different from those who are exposed to basic mathematics subjects only, is accepted. The results indicate that knowledge construction patterns do not depend on relatedness of course to mathematics. It also implies that interest in mathematics alone does not place one in an advantageous position as far as development of problem solving thinking schemata is concerned.

In Table 3, the dependence of thinking pattern categories to degree of exposure to mathematics is not significant, indicating that the distributions of mathematics subjects with respect to knowledge construction patterns is not significantly different from those who are exposed to basic mathematics subjects only, is accepted. The results indicate that knowledge construction patterns do not depend on relatedness of course to mathematics. It also implies that interest in mathematics alone does not place one in an advantageous position as far as development of problem solving thinking schemata is concerned. The two groups of respondents are not different. The result implies that more extensive experiences in mathematics is not an advantage in the formation of more productive problem solving thinking schemata.

As shown, both tests, Pearson Chi-Square and Fisher's Exact Test, yield probability values greater than 0.05 indicating that the difference in the proportions of students across groups who exhibited the same knowledge construction Table 3. Distribution according to thinking patterns and exposures to mathematics

Solution Pattern	Exposure to more math subjects	Exposure to basic math subjects only
TPG 0	12(21.29%)	16(17.58%)
TPG100	27(48.21%)	40(43.96%)
TPG200	8(14.29%)	14(15.38%)
TPG300	0	6(6.59%)
TPG400	0	2(2.2%)
TPL750	3(5.36%)	0
TPL1500	1(1.79%)	0
TPUS	5(8.93%)	13(14.29%)
Total	56	91

Pearson Chi-square: value = 12.671, exact sig. (2-sided) = 0.059 Fisher's Exact Test: Value =11.479, exact sig. (2-sided) = 0.076

patterns is well within the range of similarity. The result suggests that the knowledge and skills acquired by the students from their exposures to more mathematics subjects have not been translated into their ability to solve a non-routine or real life mathematics problem. Also, it is noted that those who exhibited knowledge construction patterns that yield the more bizarre answers which are losses of 750 and 1,500 come from those with more extensive exposures to mathematics. The expectation that students with more mathematics exposures have developed more successful knowledge construction patterns (or thinking schemata) and consequently perform better than those with limited exposure to mathematics is clearly not the case. Whether the students' more extensive exposure to Mathematics courses created in them a specific mind-set that directed their thinking to some fixed solution steps and limit the free flow of their ideas to venture on other possible paths is something that may be speculated from their works. What is certain however is that their exposures to Mathematics appear to have not been of help in facilitating the development of good problem solving thinking schemata.

In Table 4, the dependence of problem solving success to relatedness of course to mathematics and to degree of exposure to mathematics is not significant, indicating that the distributions of the three groups of respondents with respect to success categories are similar.

As shown, the values of the two tests for similarity of proportions, Pearson Chi-Square test of 3.641 with exact significance for two tailed test of 0.463 and Fisher's Exact Test of 3.642 with exact significance of 0.459, indicate that any claim for dissimilarity of success rates among the three groups of students risks about 46% chance of error which is too high to be acceptable. Thus, the hypothesis, that the distributions of the three groups of students with respect to success categories of their knowledge construction patterns are not significantly different, is accepted. The result suggests that as far as solving a non-routine or real life mathematics problem is concerned, students with more exposure to mathematics subjects do not have advantage over those with limited exposure and those whose courses are not related to mathematics. This result appears to agree with the findings of other researchers that many of the students who are expected to perform well in solving non-routine problems due to their relatively better performance in regular mathematics activities, in this case due to their more extensive exposure to mathematics, were just unsuccessful as those who are not expected to perform well, in this case, due to their limited mathematics exposure.

The result means that students' performances in solving non-routine problems are independent of their mathematics learning. Thus, if the purpose of learning mathematics as articulated in literature is to develop students' abilities to solve problems, then the results shown in all the tables above indicate that the said purpose is not being attained by the students.

The results imply that the formation of successful problem solving thinking schemata does not depend on interest in mathematics or on extent of experiences in mathematics. Thus the results corroborate with the claim of the mathematician, Polya, way back in 1962 (Yeap, 2008), that routine problem solving does not contribute to students' mental development. The results corroborate with the claims of many studies that students who are expected to perform well in problem solving are unsuccessful in solving non-routine problems.

Table 4. Success rates with respect to exposure and relatedness of course to mathematics

Group	Success rate					
	Successful		Partly successful		Not successful	
	#	%	#	%	#	%
Non-math related courses	6	8.57	31	44.29	33	47.14
Math related courses	14	15.38	46	50.55	31	34.07
Exposure to more math subjects	8	14.21	27	48.21	21	37.5
Total	28	12.9	104	47.93	85	39.17

Pearson Chi-square: value = 3.641, exact sig. (2-sided) = 0.463 Fisher's Exact Test: Value =3.642, exact sig. (2-sided) = 0.459

Finally, the results corroborate with the studies of Asman and Markowitz (2001) and Higgins (1997), as cited by Arslan and Altun (2001) that even after long years of teaching, many students are still unsuccessful in solving mathematical problem, especially non-routine problems.

CONCLUSIONS AND RECOMMENDATIONS

Students construct different knowledge patterns from the same problem situation, thus indicating the need for collaboration in order to harmonize ideas towards the production of more effective and successful thinking schemata.

Relatedness of course to mathematics and differential exposures of students to mathematics appear to be not factors that differentiate between those with successful knowledge construction patterns (or thinking schemata) and those with less successful ones.

Abilities of students to solve non-routine problems are independent of their mathematics learning.

Real life problem solving situations should be made in the context of all discussions of mathematical concepts.

Non-routine problems that can elicit divergent thinking patterns may be administered at the beginning of classes in order to know the proportion of those who already have good knowledge construction patterns (or thinking schemata) and those who do not have yet.

There is a need to enhance the existing mathematics curricula for basic education to make genuine problem solving the driving force for learning mathematics.

It is helpful to offer problem solving courses in the elementary and secondary levels where students are exposed to different types of problem solving situations, where their solving skills thinking schemata are explored and discussed and where their metacognitive abilities are developed. If students are aware of their thinking patterns or of the way they process information, then it is likely that they can self-correct or self-redirect their knowledge construction patterns towards more productive ones.

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